Additional LP Models Exercises

17. (*Production and Distribution*) A company operating in a third-world country has two plants, labeled A and B, which serve a major city, labeled C. The plants produce a product that will be shipped to C for sale. For discussion purposes, call a unit of the product an "item." The cost to ship one item between the plants and to the city is shown in the matrix below. The costs are given in $/unit.

	A	B	C
A	—	6	4
B	6	—	3
C	4	3	—

Forecasts have been made for the demand for items at the city in terms of the minimum demand (that must be satisfied), the additional demand (that may be satisfied if profitable), and the revenue per item. This information is as follows.

Month	Minimum demand	Additional demand	Revenue
1	100	50	$100
2	120	70	$150
3	140	50	$130

The company has 60 employees at each plant. Each employee can produce 2 items per month. During the first month the employees are paid a wage of $120 per month. This amount must be paid whether or not the workers produce. Because of this low pay, the company is expecting a strike in the second month. They will not pay the employees during this month so, if the plants are to operate, other workers must be hired. The company expects that it will be necessary to pay their regular employees $150 per month after the strike. The company need not rehire all the workers if it is not profitable to do so.

To increase production during month 1 and 3, and to provide some production in month 2, the company can hire teenagers. These workers are only paid $100 per month and will produce an average of $1\frac{1}{2}$ items per month. There are as many as 100 teenage workers available. Their cost and availability remains constant over the three months.

Items can be stored at the plants as inventory. The cost of storing an item for one month is $3 at plant A and $1 at plant B. Because of environmental factors, 20% of the items stored at A at the beginning of a month are ruined during the month. The corresponding loss for plant B is 40%. For simplicity assume that all production, shipments, and sales take place at the beginning of each month.

Develop a linear programming model that when solved will determine an optimum production plan for the company over the three months. The plan is to specify for each month the number of items to produce, the number of the regular workforce to hire, the number of teenagers to hire, the quantities of shipments between the plants, the quantities of shipments from the plants to the city, and the total amount sold in the city. Find the solution to this problem.

18. The main product of a company is an item that includes an expendable replacement cartridge. Call the main product *A*. Call the expendable cartridge *B*. Each unit of *A* sold requires one unit of *B*. The life of the cartridge is one month so the replacement sales of *B* in any one month will be equal to the number of items *A* sold in prior months. The company is now the only manufacturer of *A* and *B* but competition will make both products unprofitable after one year.

It is now January 1 and the company is planning its production of *A* and *B* for the coming 12 months. One thousand units of *A* have already been sold. The table below shows the maximum sales of *A* that can be expected during each of the next 12 months. As can be seen, the unit selling price of *A* declines during the year. The selling price of *B* is constant over the year at $50 per unit. The company can sell less than the maximum for *A* but no more. Sales of B depend entirely on previous sales of *A*. The company is obligated to produce all of product *B* that is demanded.

Maximum sales and selling price for product *A*

	Jan	Feb.	Mar.	April	May	June
Maximum sales	500	700	1000	1200	1500	1700
Selling price	$500	450	425	400	375	350

	July	Aug.	Sept.	Oct.	Nov.	Dec.
Maximum sales	1500	1200	1000	8000	800	800
Selling price	325	300	275	250	250	250

Both products can be placed in inventory for later sale. The cost of storage is $3/unit/mo for *A* and $1/unit/mo for *B*. There is no initial inventory and no inventory is planned for the end of the year.

Each unit of *A* requires eight hours of labor and each unit of *B* requires one hour of labor. Each worker contributes 100 hours of labor per month. The company currently has 20 workers who will be employed continually throughout the year. Additional workers may be hired with a wage of $2000 per worker per month. Because of training time new workers will not contribute to production during their first month on the job. Any workers hired are kept at least until the end of the year. The current workers do not have to be trained.

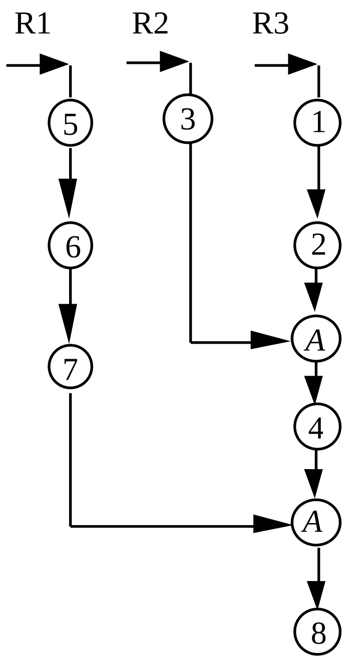
Both products are made from a single raw material. Product *A* requires 5 pounds per unit and product *B* requires 2 pounds per unit. In any one month 10,000 pounds of the raw material is available at a cost of $8 per pound. Any additional amount may be purchased of $11 per pound. Raw material must be purchased in the month it is used.

For simplicity, assume that all transactions such as production, hiring of workers, purchases of raw materials, and additions or removals from inventory occur at the beginning of the month. Set up and solve a linear program that determines an optimum production schedule, workforce plan, and raw material purchase plan for the company.

19. The diagram below shows the operations performed in the manufacture of a product. Raw materials labeled R1, R2, R3 are introduced at nodes 1, 3 and 5, and are processed through operations represented by the remaining nodes. The arrows in the diagram define precedence relations; e.g., operation 1 must be performed before operation 2. Each operation requires an expenditure of time as indicated in the accompanying table. During an operation, a fraction of the items being processed are ruined. This is indicated in the *loss* column of the table. The damaged items are removed at the output of the corresponding operation. If x units enter an operation that has loss w, then $x(1 - w)$ leave the operation.

    The nodes labeled *A* represent assembly operations, where subassemblies of the product are joined to form complete or partially completed units. Thus at the predecessor of node 4, subassemblies produced in the branch consisting of operations 1 and 2 are combined with subassemblies produced in the branch consisting of operation 3. A characteristic of assembly nodes is that the flow of each subassembly entering the node must equal the flow of combined assemblies leaving the node, minus any losses.

    Each operation is performed by one of five machines. The machine assignments are given in the table. For example, operations 1, 3 and 5 are all performed on machine 1. It is assumed that one hour of labor is required for each machine hour.



Operation	Loss	Operation time (hrs)	Machine assignment	Raw material
1	0.1	1.0	1	1kg of R3
2	0.2	2.0	2	---
3	0.1	0.5	1	2kg of R2
4	0.2	0.3	3	---
5	0.1	0.4	1	3kg of R1
6	0.1	2.0	2	---
7	0.3	0.5	4	---
8	0.1	1.2	5	---

Defining x_i as the product flow entering operation i for $i = 1,\ldots,8$, and v as the finished product flow leaving node 8, the following equations give the relations between the flows at the operations.

$$0.9x_1 - x_2 = 0 \quad (1)$$

$$0.8x_2 - x_4 = 0 \quad (2)$$

$$0.9x_3 - x_4 = 0 \quad (3)$$

$$0.8x_4 - x_8 = 0 \quad (4)$$

$$0.9x_5 - x_6 = 0 \quad (5)$$

$$0.9x_6 - x_7 = 0 \quad (6)$$

$$0.7x_7 - x_8 = 0 \quad (7)$$

$$0.9x_8 - v = 0 \quad (8)$$

Set up and solve the appropriate linear programming model for the situations described below. Parts (a), (b) and (c) are cumulative in that new information is added about the situation in each part. Part (d) is independent of the other parts.

a. There are three machines of each type available, each operating 60 hr/week. A total of 300 labor hours are available for use on the product. Find the production plan that maximizes the finished products assembled in a week.

b. In addition to the information in part (a), raw material 1 cost $5/kg, raw material 2 cost $3/kg, and raw material 3 cost $10 for the first 20 kg and $12 for each kg. greater than 20. Up to 20 units of the product can be sold for $60/unit, while any amount between 20 and 50 can be sold for $45/unit. Amounts greater than 50 can be sold for $30/unit. The labor cost is $4 hr. Find the production plan that maximizes the profit for this product.

c. We are now provided with the opportunity to purchase more machines. The table below shows the weekly amortized cost and the space required for each machine. A total of 1000 square meters of new space is available.

	Machine				
	1	2	3	4	5
Cost ($/wk)	200	100	175	130	300
Space (sq m / machine)	10	20	15	25	10

We also learn that the subassemblies produced by operations 1, 2, 3 and 4 can be purchased outside the company for $17 each. We want to have both the purchase option and the production option included in the model. Find the production plan, machine purchase plan, and outside purchase plan that maximizes the profit for this product.

d. This part is separate from parts (a) – (c) but uses the information from the problem statement. There are three machines of each type available, each operating 60 hours per week. A manager interested in efficiency says that the most important measure is the use of machines. He wants the individual machines to be as busy as possible and so proposes the new objective of maximizing the time spent on the least used machine. Write and solve the linear programming model that maximizes the minimum of the machine usage times. Is this a good objective for the company? Explain.

20. A company makes four grades of gasoline from a mixture of three types of crude oil. The characteristics of the oil and the requirements for each gasoline type are shown in the first table below. The octane ratings for gasoline grades are minimums and the percentage of lead content are maximums. There are no losses in the process and one barrel of crude mixture produces one barrel of gasoline.

Crude Oil Characteristics

Ingredient	Octane rating	Lead content (%)	Cost per barrel ($)

Crude 1	95	0.8	45
Crude 2	85	2.1	30
Crude 3	75	3.4	25

Gasoline Requirements

Grade	Octane rating	Lead content (%)	Revenue per barrel ($)
Regular	85	2.0	37
No lead	81	1.0	35
Premium	85	1.2	40
Super	90	1.2	43

For the current month the company must purchase at least 2000 barrels of each crude type because of contracts with suppliers. An additional 3000 barrels may be purchased for the price shown in the table. Any additional amounts may be purchased on the spot market for 1.2 times the prices in the table. The company can afford to spend no more than $200,000 for spot purchases.

Contracts with distributors require that at least 2000 barrels of each gasoline grade be produced. Additional quantities of all products can be sold at the given revenues shown in the second table up to a total of 10,000 barrels (this figure includes the contracted amount). The company is planning a special promotion to sell the *Super* brand under a different name. As much as 7000 additional barrels of *Super* can be sold in this way but the price of the additional amount will be only $39 per barrel. The management wonders whether the special promotion is a good idea. The total production capacity of the plant for all grades of gasoline is 30,000 barrels per month.

a. Write and solve the linear programming model that will determine the optimum production plan.

b. From the solution, how can you tell if the promotion regarding *Super* is a good idea?

c. The company is considering increasing plant capacity (barrels per month). What information can you get from your solution to help guide this decision?