Ch. 9 Nonlinear Programming Models Additional Exercises

25. A firm stocks n different items in the same warehouse. Item i requires a_i square feet of warehouse space per unit. The total warehouse space available is A square feet. All n items are replenished independently in batches of size Q_i. Holding costs assessed on the average inventory level amount to h dollars per dollar invested per year. The value of item i is v_i dollars per unit. The fixed ordering cost for item i is K_i per batch, and the annual demand is d_i.

a. Develop a mathematical program to minimize the annual total inventory holding and ordering costs for all n items subject to warehouse availability. Assume that each item is allotted a space in the warehouse required to store Q_i.

b. Given that the warehouse space available is A and all will be used, form the Lagrangian function and determine expressions to find the optimal order quantities Q_i^* (i.e., set partial derivatives to zero).

c. Use $A = 2000$ sq ft, $h = 0.2$, and the following data to find the optimal Q_i^* and the minimum cost. How much of the total cost can be attributed to the warehouse restriction?

Parameter	$i =$	1	2	3
d_i (units)		12,000	5,000	2,000
v_i ($/unit)		$20	$40	$50
K_i ($/order)		$160	$200	$50
a_i (sq ft)		1	2	4

26. A firm produces n items on the same machine that operates 250 days per year. Each item is produced in batches of size Q_i at at a rate b_i per day. For every batch of item i produced, the setup time is a_i days. Let d_i be the annual demand, v_i the product value per unit, h the holding cost per dollar invested per year, and K_i the fixed setup cost per batch for the item i.

a. Find a general expression for the total annual cost and express the constraints on the production capacity mathematically.

b. Find expressions that, given data, would allow you to determine the optimal constrained batch sizes Q_i.

c. Using common sense, discuss why this model is only a suitable approximation if the number of items n is relatively large, and why it could not be implemented in n were small, say, 2, 3 or 4.

27. Machine I produces a certain item at a uniform rate of 960 for a 480-minute day, and machine II at a uniform rate of 1440 per day. The item has a market value of $2 and can be sold immediately after manufacture. Proper adjustment of the machines is critical, and the number of defective parts produced is dependent on the number of adjustments made per day. Past experience shows that the number of defective parts produced by machine I is $256/N$, where N is the number of adjustments made on machine I, and $125/M$ for machine II, where M is the number of adjustments made on machine II.

Each part produced has a material cost of $0.80, regardless of whether it is salable or defective. Defective parts cannot be sold and have no scrap value. Each adjustment has a cost of $4 for machine I and $6 for machine II, and takes 10 minutes during which time no production occurs. A fractional number of adjustments per day simply means that the days are not necessarily started with an adjustment, but that an adjustment is made after $960/N$ or $1440/M$ parts are produced. However, because of manpower restrictions no more than 6 adjustments can be made daily. The firm wishes to maximize daily profits. How should it operate the two machines?

Let $P(N, M)$ be the difference between revenues and costs per day. Write the mathematical program that minimizes $P(N, M)$ and solve with appropriate software.

28. (*Car pooling*) A total of m persons have filled out a questionnaire for the purpose of forming car pools. Items on the questionnaire included name, address, sex, occupation, location of work, time work begins, and time work ends. Based on this information, analysts have determined a compatibility index between person i and person j, denoted by c_{ij}, for all pairs of persons. A small value for this index implies incompatibility.

Let U and L represent the maximum and minimum number of people in a car pool, respectively. Also, let

$$x_{ik} = \begin{cases} 1 & \text{if person } i \text{ is assigned to car pool } k \\ 0 & \text{otherwise.} \end{cases}$$

The objective is to construct n car pools so that the associated total compatibility index is maximized and each person is assigned to one of the vehicles.

a. Using the above notation write out the objective function that is to be maximized.

b. Write all the constraints (using linear functions only).

c. Write a linear constraint that would assure person 1 and person 2 are not assigned to the same vehicle.

d. Write a linear constraint that assures persons 3 and 4 are assigned to the same vehicle.

Define any additional notation used.